Building Option Price Index

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Abstract

In this paper, I use real data in building call and put option price indices for 30 US blue-chip stocks both individually and collectively, by applying divisor-adjusting methodologies (mimic methods used for DJIA Index) in order to achieve continuity, comparability and consistency of the indices. The result shows that it is feasible. As horizontal measures, these indices provide better timing gauges to follow option price movements for the underlying assets, while implied volatilities could provide vertical measures to identify which options are under- or over-valued. With option price indices and implied volatilities, investors could make better and informed investment decisions.

Keywords:

Option Price Average Index, Divisor Adjusting Methodology, Implied Volatility, Annualized Volatility, Benchmark Option Price Index

JEF Classifier: C02, G00

1 INTRODUCTION

Currently there is no index for call and put option prices movements over the time horizon. I do believe that investors could make more-informed and better decisions if there are option prices indices available such as Options Price Average Indices (OPAI) as benchmarks for their decisions-making.

Based on Black-Scholes Model, the option price is a function of the current price of the underlying asset (S), the strike price (K), the volatility of the underlying asset price (σ), the time to maturity (T), the risk-free rate (R) and the dividend yield (Q) (if have). Because of several independent variables used in the option price function, many different options with different prices can be written on the same underlying asset, and will expire within limited time in the future, and many new options will be created periodically on the new market conditions and exist only for certain time period. To track the options prices movement over the time for an underlying asset, we need to treat all options available for the time being collectively as the constituents to construct an option index to follow their price movements. Like Dow Jones Industrial Average Index, by applying divisor-adjusting-methodologies (DAM), it is feasible to build a valid and consistent option price index on a continuous time base. No matter how many new options will be written at new prices on the new market conditions for an underlying asset; no matter how many options for the same underlying asset are going to expire, this DAM method will allow us to eliminate all biases to keep the option index of validities. Go a further step, given options prices indices for all required underlying assets such as all constituent stocks of Dow Index, we will be able to build a benchmark option price index by treating all individual options prices indices collectively as constituent indices (Index-on-Indices). The key tool, divisor-adjusting-methodology, for building option price indices, will be illustrated in details in the following section.

2 USING DIVISOR-ADJUSTING METHODOLOGY (DAM) IN BUILDING OPTION PRICE AVERAGE INDEX (OPAIS)

Let's make a simple sample to illustrate the divisor-adjusting-methodology. Supposed that an underlying asset (S) has only 3 different option (A, B, C) at time T, and one of them (B) will expire at time T+1, and 2 new options (E and F) will be created at time T+1. All are equally-weighted, and the prices are shown in ***Table 1***:

Table 1

Option Price	A	B	C	E	F	Num(N)	Total (S)	Avg	Adjusted Divisor (d)	Index
T	1.50	2.00	2.50			3	6.00	2.00	1.00	**2.00**
T+1	1.50		2.50	5.00	1.00	4	10.00	2.50	1.25	**2.00**

Because the prices of the remaining options (A and C) unchanged, the option price average index should remain the same, no matter what prices for these 2 new options (E and F). Assuming the adjusted divisor (d_t) equal to 1, at the time T the index (I_t) is computed by the price average divided by the adjusted divisor (d_t), or that

$$I_t = \frac{S_t}{d_t N_t} = \frac{6}{1(3)} = 2$$

Similarly, at the time T+1 the index (I_{t+1}) is calculated as follows:

$$I_{t+1} = \frac{S_{t+1}}{d_{t+1} N_{t+1}} = \frac{10}{1.25(4)} = 2$$

Here the adjusted-divisor is determined by the following formula[1]:

$$d_{t+1} = d_t \frac{S_t^{(a)}}{S_t} / \frac{N_{t+1}}{N_t}$$

Here, $S_t^{(a)} = S_t - \sum (Expired)|_{(t)} + \sum (New)|_{(t+1)}$

So

$$S_t^{(a)} = 6 - 2 + (5+1) = 10$$

$$d_{t+1} = (1)\left(\frac{10}{6}\right) / \left(\frac{4}{3}\right) = 1.25$$

Let's change the prices of the new option (E an F), say prices of them are 2.00 and 10.00 respectively (see ***Table 2***).

Table 2

Option Price	A	B	C	E	F	Num(N)	Total (S)	Avg	Adjusted Divisor (d)	Index
T	1.50	2.00	2.50			3	6.00	2.00	1.00	**2.00**
T+1	1.50		2.50	2.00	10.00	4	16.00	4.00	2.00	**2.00**

So it just needs to change the adjusted divisor to 2.00 as calculated as follows:

$$d_{t+1} = d_t \frac{S_t^{(a)}}{S_t} / \frac{N_{t+1}}{N_t} = (1)\left(\frac{16}{6}\right) / \left(\frac{4}{3}\right) = 2.00$$

$$I_{t+1} = \frac{S_{t+1}}{d_{t+1} N_{t+1}} = \frac{16}{2(4)} = 2.00$$

From above, we could also see that no matter how the prices of the newly created options in the new market conditions are, they will not lead to any biases for the index's continuity and consistency by adjusting the divisor. Of course, when A or C and or both change prices, the index could change accordingly.

To generalize the calculation of the adjusted divisor for a weighted price index, it is assumed that:

[1] See mathematical derivation Appendix 1: Proof (1)

(i) The prices will be weighted by something, and let's called it "volume";
(ii) Total number of different options at time T is $N_{(t)}$, of which $N_{(r, t)}$ options remain at time T+1 and N(e, t) expires at time T+1
(iii) At time T+1, $N_{(n, t+1)}$ new options will be created for the underlying asset with total new prices and volumes.
(iv) All prices and their volumes for the $N_{(r, t)}$ remaining options unchanged;
(v) Options with different maturities are treated equally in calculations;
(vi) The adjusted divisor (d_1) for the first day of launching the index always set to 1.

Because all prices and their volumes for the $N_{(r, t)}$ remaining options are unchanged, the weighted price index should be remaining the same from time T to T+1. Since at time t the weighted price average index (WPAI) is computed as the weighted price average (WPA_t) divided by the adjusted divisor (d_t), or that,

$$WPAI_t = \frac{WPA_t}{d_t} = \frac{\sum_{i=1}^{N_t} V_{(i,t)} P_{(i,t)}}{d_t \sum_{i=1}^{N_t} V_{(i,t)}}$$

Let $\sum_{i=1}^{N_t} V_{(i,t)} P_{(i,t)} = S_t, \quad \sum_{i=1}^{N_t} V_{(i,t)} = V_t$

So $WPAI_t = \frac{S_t}{d_t V_t}$

At time T+1, we could simply make adjustment for the divisor by the following formula[2]

$$d_{t+1} = d_t \left(\frac{S_t^{(a)}}{S_t} \right) / \left(\frac{V_t^{(a)}}{V_t} \right)$$

Here,

$$S_t^{(a)} = S_t - \sum_{j=1}^{N(e,t)} V_{\exp ired(j,t)} P_{epired(j,t)} + \sum_{j=1}^{N(n,t+1)} V_{new(j,t+1)} P_{new(j,t+1)}$$

$$V_t^{(a)} = V_t - \sum_{j=1}^{N(e,t)} V_{\exp ired(j,t)} + \sum_{j=1}^{N(n,t+1)} V_{new(j,t+1)}$$

With this adjustment, the option price average index will not change, or that,

$$WPAI_{t+1} = \frac{WPA_{t+1}}{d_{t+1}} = \frac{\sum_{i=1}^{N_{t+1}} V_{(i,t+1)} P_{(i,t+1)}}{d_{t+1} \sum_{i=1}^{N_{t+1}} V_{(i,t+1)}} = \frac{\sum_{i=1}^{N_t} V_{(i,t)} P_{(i,t)}}{d_t \sum_{i=1}^{N_t} V_{(i,t)}} = \frac{WPA_t}{d_t} = WPAI_t$$ [3]

[2] See mathematical derivations in Appendix 1: Proof (2)
[3] See mathematical derivations in Appendix 1: Proof (2)

By applying this divisor methodology (DAM) on a daily base, we will able to achieve the continuity, comparability and consistency for the options price average indices (OPAI) on a daily fashion, no matter how many different options created periodically for the same underlying asset and no matter at what prices they created as new options in new market conditions, and no matter when they expires.

Therefore, we can build option price average indices (OPAI) for both call option price average indices (COPAI) and put option price average (POPAI) for any kind of underlying assets.

The above section is illustrated to build Options Price Average Indices (OPAI) for individual underlying asset. Go a further step, for Benchmark Options Price Average Indices (BOPAI), firstly we can calculate option price indices for all constituent underlying assets consisting a benchmark index such as Dow Jones Industrial Average, S&P500, Nasdaq 100, etc., then simply take the average (equally-weighted) or weighted average of the option price indices for all constituent underlying assets to create BOPAIs (like "average-on-average" or "Index-on-Indices").

3 OPTION PRICE AVERAGE INDEX (OPAIS) FOR ALL 30 CONSTITUENT STOCKS OF DJIA INDEX (FEBRUARY 1, 2005 TO MARCH 31, 2008) BY USING REAL DAILY CLOSE OPTIONS DATA

Now I use the real option daily close data (Feb. 1, 2005 –Mar. 31, 2008)[4] for all stocks of DJIA Index, by applying the above methods and processes to calculate Call Option Price Average Indices (COPAI) and Put Option Price Average Indices (POPAI) for all of these underlying assets based on the following assumptions:

(i) when an option has no transaction during a day, it is assumed that its option price unchanged during that day;

(ii) when a newly created option has no transaction at the very beginning, which has no price or zero price, in this case it is deemed as not existing, and will not be taken into calculations until it has the first transaction.

I just show the results in graphs (*Chart 1-12*) for 12 stocks (underlying assets) with a summary beneath each chart here for illustration purposes. These stocks' Ticks include AA, AIG, C, DD, GE, HD, IBM, JPM, KO, MSFT, PFE, WMT. For making an easy vision comparisons, the charts for daily close prices[5] of all of these stocks during the same period also illustrated together.

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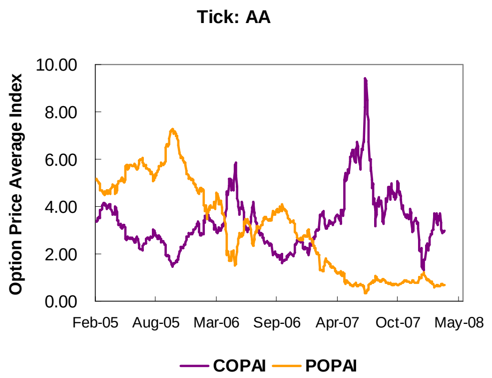
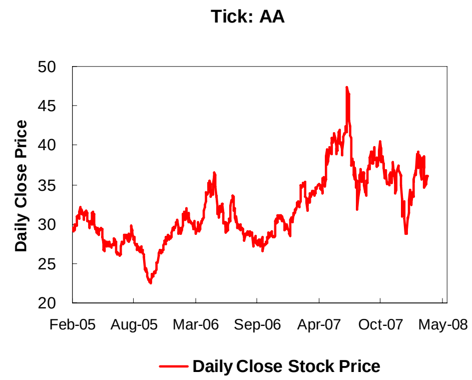
[4] Data source: www.stricknet.com

[5] Data source: Yahoo Finance

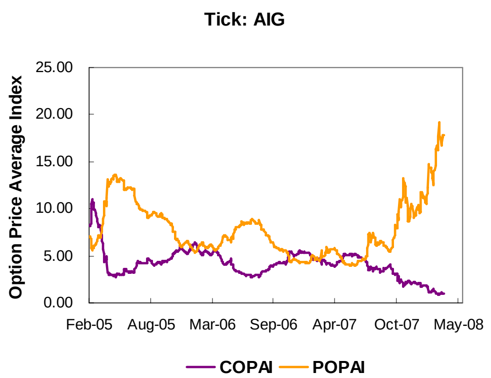
Chart 1: Tick-AA

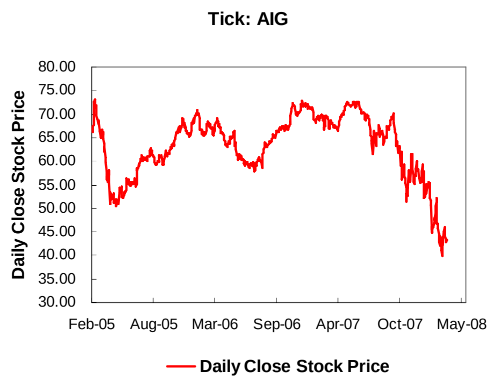




Alcoa: Option Price Average		
Type	**Call**	**Put**
Historical[6] Low (Date)	$1.32 (08/13/2008)	$0.35 (07/16/2007)
Historical high (Date)	$9.43 (07/13/2007)	$7.28 (10/13/2005)
52 weeks range	$1.32-$9.43	$0.35-$1.35
Currently[7] at	$2.97	$0.69

Chart 2: Tick- AIG

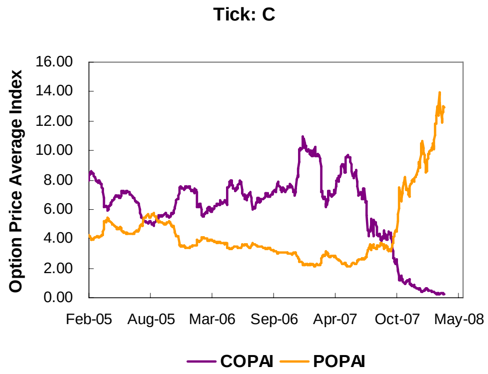


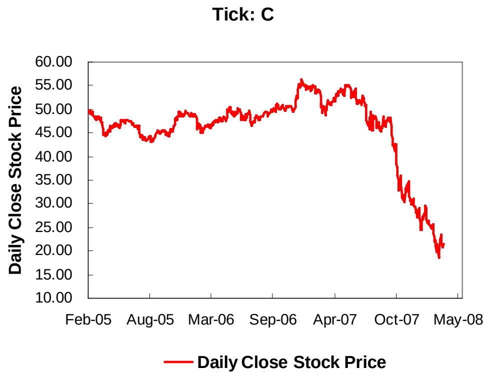


AIG: Option Price Average		
Type	**Call**	**Put**
Historical Low (Date)	$0.82(03/17/2008)	$3.97 (06/18/2007)
Historical high (Date)	$11.04 (02/11/2005)	$19.13 (03/17/2008)
52 weeks range	$0.82-$5.20	$3.97-$19.13
Currently at	$0.94	$17.83

[6] Hereafter "historical" refers to the period of Feb.1, 2005 to Mar.31, 2008
[7] Hereafter "currently" refers to the date of March 31, 2008

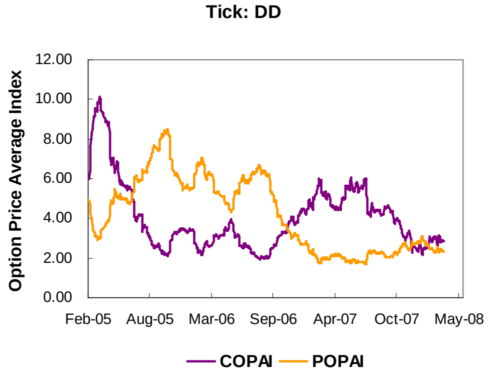
Chart 3: Tick-C

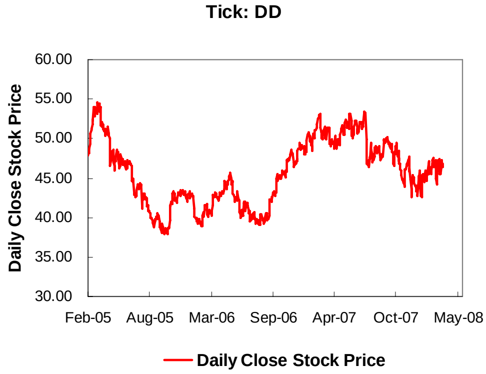




CITIGROUP: Option Price Average		
Type	**Call**	**Put**
Historical Low (Date)	$0.22 (03/17/2008)	$2.10 (05/30/2007)
Historical high (Date)	$10.95 (12/27/2006)	$13.96 (03/17/2008)
52 weeks range	$0.22-$9.67	$2.10-$13.96
Currently at	$0.27	$12.90

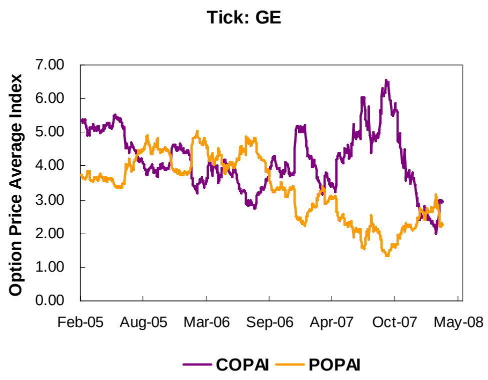
Chart 4: Tick-DD

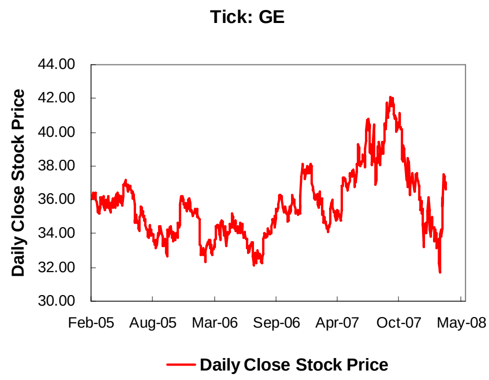




DuPont: Option Price Average		
Type	**Call**	**Put**
Historical Low (Date)	$1.95 (08/14/2006)	$1.67 (07/23/2007)
Historical high (Date)	$10.15 (03/09/2005)	$8.51 (10/17/2005)
52 weeks range	$2.18-$6.06	$1.67-$3.10
Currently at	$2.87	$2.32

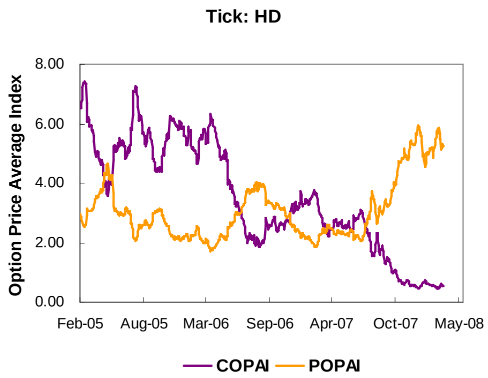
Chart 5: Tick-GE

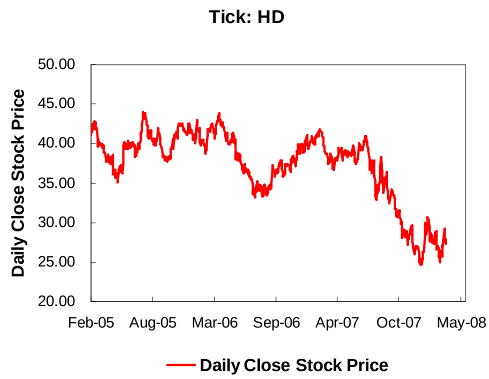




GE: Option Price Average		
Type	**Call**	**Put**
Historical Low (Date)	$2.00 (03/10/2008)	$1.34 (10/10/2007)
Historical high (Date)	$6.55 (10/02/2007)	$5.05 (02/07/2006)
52 weeks range	$2.00-$6.55	$1.34-$3.15
Currently at	$2.96	$2.27

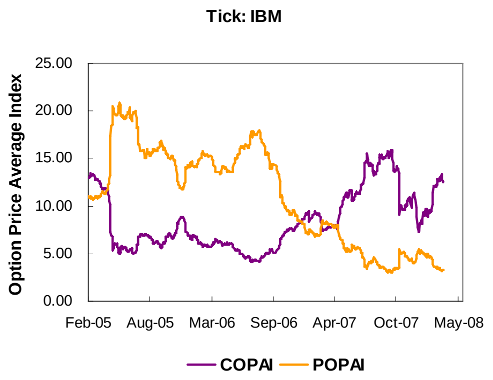
Chart 6: Tick – HD

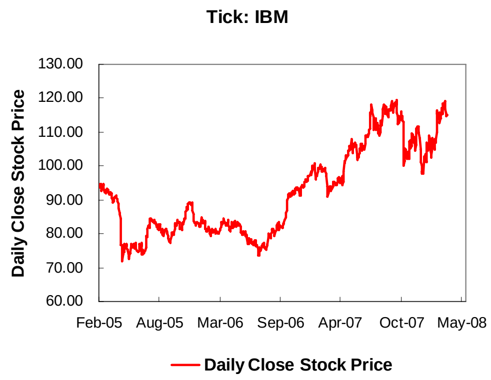




Home Depot: Option Price Average		
Type	**Call**	**Put**
Historical Low (Date)	$0.46 (03/10/2008)	$1.72 (03/23/2006)
Historical high (Date)	$7.41 (02/17/2005)	$5.94 (01/09/2008)
52 weeks range	$0.46-$3.10	$2.06-$5.94
Currently at	$0.55	$5.26

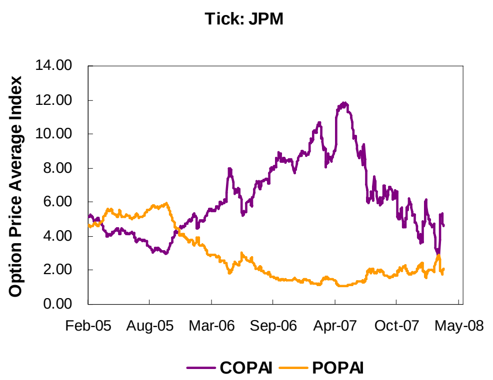
Chart 7: Tick – IBM

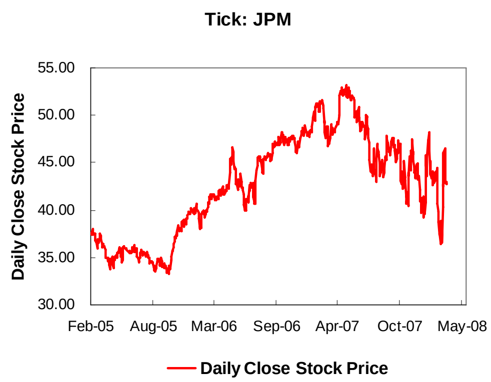




IBM: Option Price Average		
Type	**Call**	**Put**
Historical Low (Date)	$4.12 (07/18/2006)	$3.02 (10/16/2007)
Historical high (Date)	$15.92 (10/16/2007)	$20.85 (05/13/2005)
52 weeks range	$7.23-$15.92	$3.02-$8.02
Currently at	$12.54	$3.34

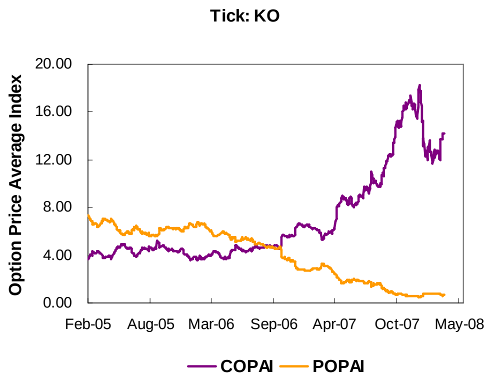
Chart 8: Tick-JPM

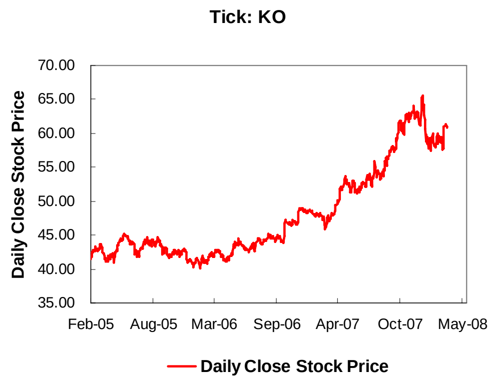




JP Morgan: Option Price Average		
Type	**Call**	**Put**
Historical Low (Date)	$2.82 (03/14/2008)	$1.07 (05/16/2007)
Historical high (Date)	$11.85 (05/09/2007)	$5.95 (10/13/2005)
52 weeks range	$2.82-$11.85	$1.07-$2.94
Currently at	$4.59	$2.03

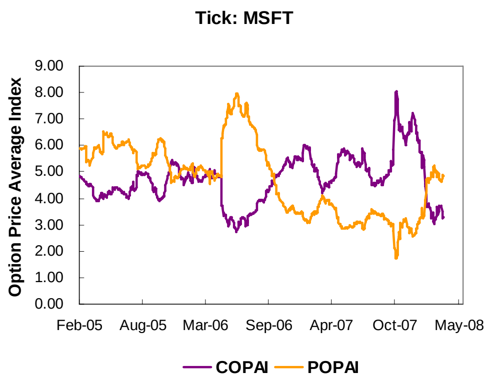
Chart 9: Tick - KO

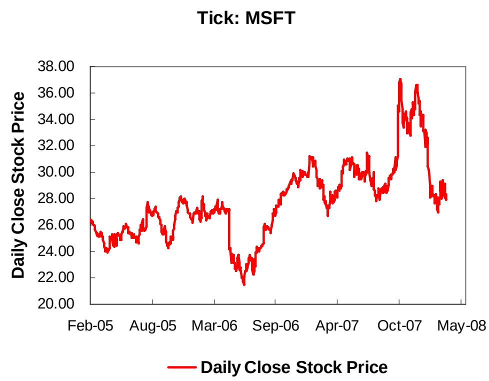




Coca-Cola: Option Price Average		
Type	**Call**	**Put**
Historical Low (Date)	$3.54 (01/20/2006)	$0.48 (01/10/2008)
Historical high (Date)	$18.29 (01/10/2008)	$7.30 (02/02/2005)
52 weeks range	$5.77-$18.29	$0.48-$2.69
Currently at	$14.20	$0.63

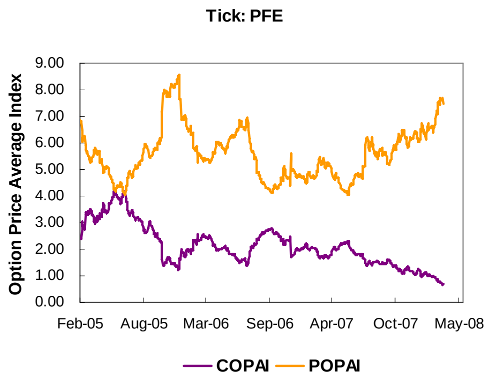
Chart 10: Tick - MSFT

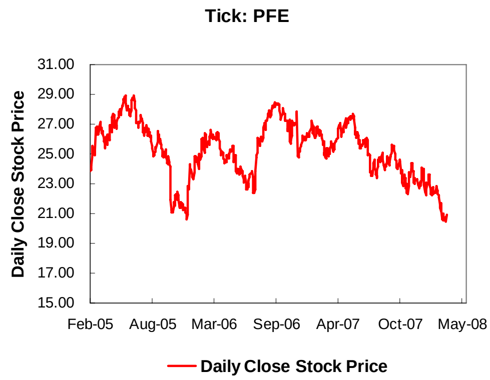




Microsoft: Option Price Average		
Type	**Call**	**Put**
Historical Low (Date)	$2.74 (06/13/2006)	$1.72 (01/11/2007)
Historical high (Date)	$8.05 (02/11/2007)	$7.97 (06/13/2006)
52 weeks range	$3.03-$8.05	$1.72-$5.23
Currently at	$3.31	$4.85

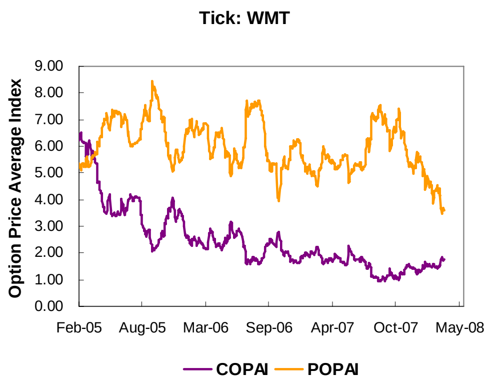
Chart 11: Tick – PFE

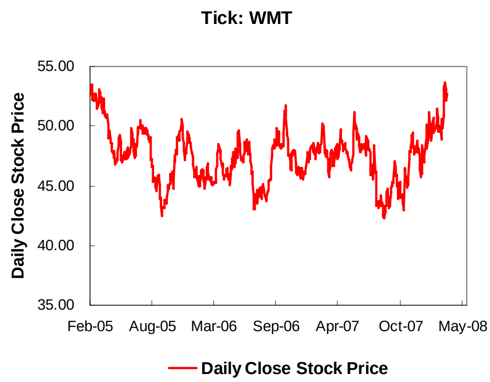




Pfizer: Option Price Average		
Type	**Call**	**Put**
Historical Low (Date)	$0.67 (03/28/2008)	$4.04 (06/04/2007)
Historical high (Date)	$4.22 (05/24/2005)	$8.55 (12/12/2005)
52 weeks range	$0.67-$2.30	$4.04-$7.69
Currently at	$0.69	$7.50

Chart 12: Tick - WMT





Mal-Mart: Option Price Average		
Type	**Call**	**Put**
Historical Low (Date)	$0.94 (09/10/2007)	$3.48 (03/24/2008)
Historical high (Date)	$6.52 (02/08/2005)	$8.42 (09/21/2005)
52 weeks range	$0.94-$2.28	$3.48-$7.53
Currently at	$1.76	$3.60

4 CORRELATIONS AND VOLATILITIES OF OPTION PRICE INDICES AND UNDERLYING ASSETS

After we have obtained all option price average indices for these underlying assets, correlations coefficients between the call option average index (COPAI) as well as the put option average index (POPAI) and the underlying asset price during the same time period have been calculated. The results show that COPAIs are highly positively correlated with their underlying stock prices with average of 0.8264, while POPAIs are on opposite, highly negatively correlated with their underlying stock prices on average of -0.8007, summarized in *Table 3 (See the details in Appendix 2)*

Table 3: **Correlation Coefficient**

Correlation Coefficient	**COPAI – Underlying Stock Price**	**POPAI – Underlying Stock Price**
Min	0.3055	-0.9774
Max	0.9898	0.2629
Average	0.8264	-0.8007

These are very good results, as call option prices theoretically increase with the underlying price's going up, while put option prices work in the opposite way. These results also affirmed that the methods using in building the options price average indices are valid and effective.

The volatilities of options price indices and the underlying asset prices have also been computed, and the results for the annualized[8] volatilities for both call option average index and put option average index as well as underlying stock prices are summarized in *Table 4* (See the details in *Appendix 2*).

Table 4: Annualized Volatilities Comparisons

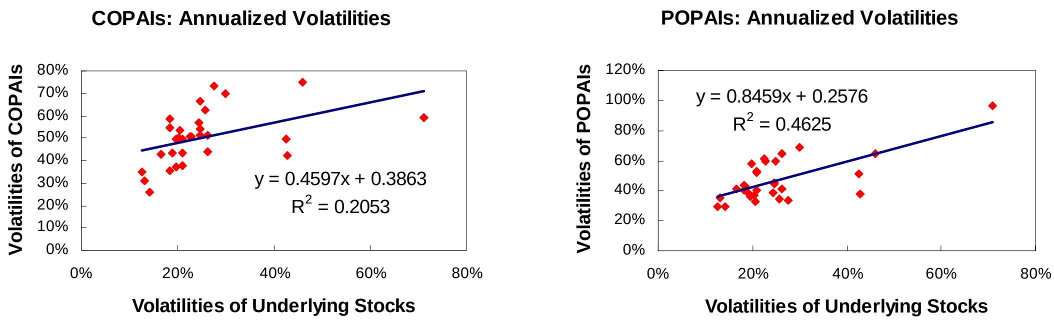
Annualized Volatilities	**COPAI**	**POPAI**	**Underlying Assets**
Min	26.1%	29.1%	12.46%
Max	75.0%	96.5%	71.00%
Average	50.2%	47.0%	25.20%

Both call and put option price average index are much more volatile (nearly double on average) than the underlying asset, which explains leverage effects for options investments.

Next by linearly regressing the correlation coefficients and volatilities for both COPAIs and POPAIs with respect to the volatilities of the underlying asset prices, I graph in Chart 13.

[8] The annualized volatility is computed as $\sigma_{Annualized} = \sigma_{daily}\sqrt{250}$, and the daily volatility is the standard deviation of the daily returns, which is calculated by $r_t = \ln(S_t / S_{t-1})$. Here S_t is the daily close price for underlying asset, or the option price average index at time (t).

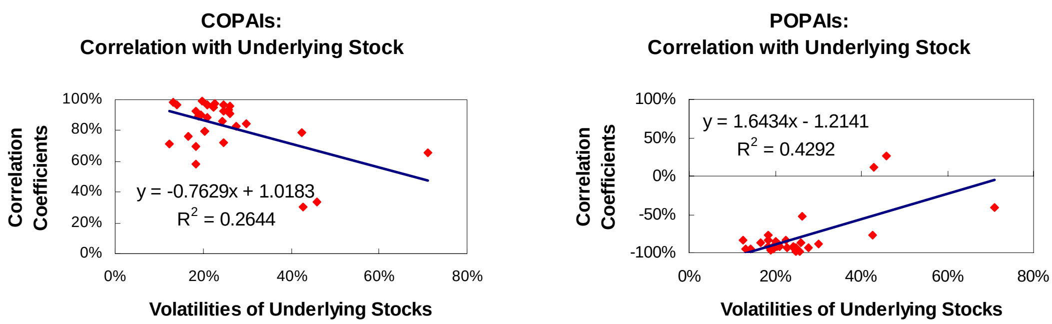
Chart 13: Regression on Volatilities



Although R-squares are relatively low (0.2053 for COPAIs, and 0.4625 for POPAIs with underlying assets), it is apparently that volatilities for both COPAIs and POPAIs are positively correlated with the volatilities of the underlying assets, because higher volatility of an underlying asset could result in higher volatility for its options prices index. The regression coefficients for both COPAIs and POPAIs are less than 1, which could reasonably explain that on average both call and put options price changes should be less than the changes of the underlying asset prices. It seems true, since both call and put options price changes would most likely be less than the price changes of the underlying asset.

Similarly, by regressing correlation for both COPAIs and POPAIs with respect to the volatilities of the underlying assets, the graphs are shown in *Chart 14.*

Chart 14: Regression on Correlation



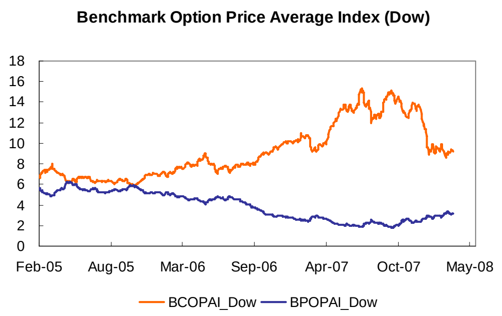
It seems that higher volatilities of the underlying asset tend to reduce the positive correlation between call option price average index and the underlying asset price movement. It could be true because higher volatility of the underlying asset leads to higher volatility of call option index, but not to the same extent, which causes lower positive correlation coefficient for call option index. It is also true for the case of the put option price index, because it weakens the negative correlation (i.e., reducing the correlation coefficient in absolute value).

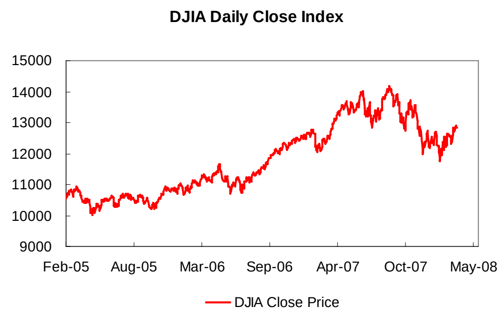
5 BENCHMARK OPTION PRICE AVERAGE INDEX FOR DJIA INDEX (FEBRUARY 1, 2005 TO MARCH 31, 2008)

The Benchmark Option Price Average Index (BOPAI) is designed for a collection of the underlying assets such as all constituent stocks of Dow Jones to follow their options prices changes collectively over the time. BOPAI is simply set equal to the arithmetic (equally weighted) or weighted (such as market-cap weigheted) average of option price indices for all constituent stocks, which construct the benchmark index such as DJIA (like "Average-on-Average"). BOPAI is also classified as BCOPAI (Benchmark Call Option Price Average Index) and BPOPAI (Benchmark Put Option Price Average Index).

Here, just BOPAI for DJIA Index has been constructed, as BCOPAI.Dow and BPOPAI.Dow defined as a Dow Benchmark Call Option Price Average Index and Dow Benchmark Put Option Price Average Index respectively. Again based on COPAIs (Call Option Price Average Indices) and POPAIs (Put Option Price Average Indices) for all 30 Dow stocks obtained, I simply take an arithmetic average of them to calculate BCOPAI.Dow and BPOPAI.Dow Indices during the period of Feb. 1, 2005 to Mar. 31, 2008. The results are graphed in *Chart 15*.

***Chart 15:* Benchmark Option Price Index**





The outcome shows that BCOPAI is nearly perfectly positively correlated with DJIA during the same time period, with a correlation coefficient up to 0.9765, and BPOPAI is also nearly perfectly negatively correlated with DJIA with a correlation coefficient of -0.9792. This also could provide an evidence for validity and effectiveness for the methodology I use to build the options price average indices above.

The annualized[9] volatility for DJIA during the period is calculated as 12.9%, while for BCOPAI index the annualized volatility is 23.3% and for BPOPAI is 22.5%, which are nearly double of DJIA's.

Similarly, by using the same process, Benchmark Call and Put Option Price Average indices can be easily established for other major global stock indices such as S&P Indices series, NASDAQ Index, etc..

[9] The same method as described in footnote [8] is used to calculate volatilities.

6 CONCLUSION

It is feasible to establish call and put option price average indices both for individual underlying asset and for a collection of underlying assets such as all constituent stocks for a benchmark index (such as DJIA, S&P500, etc.) by using the above processes. The divisor adjusting methodology (DAM) allows us to eliminate biases arising from change constituent options in order to maintain continuity, comparability and consistency of the option price indices. The above results obtained from the real option data during February 1, 2005 to March 31, 2008 have further provided enough proofs for the feasibility in building both call and option price average indices. The option price average indices illustrated in this paper is using an “equally-weighted average” method (or arithmetic average), and of course, it could also be possible to create options price indices in a “non-equally-weighted average” such as market-cap-weighted average to build benchmark option price indices.

With the valid and consistent option price indices, it will be great useful for investors as well as all financial world for their more-informed decision-making and researches as they could easily track and follow option price changes or movements horizontally, while they could scan options vertically by using implied volatilities to identify which options are undervalued or overvalued.

VI. REFERENCES

(i) *Financial Risk Manager Handbook (GARP)*, Philippe Jorion, 2007

(ii) *The Complete Guide to Capital Markets for Quantitative Professionals*, Alex Kuznetsov, 2006

(iii) *Statistical Analysis of Financial Data in S-Plus*, Rene A. Carmona, 2004

(iv) *Standard & Poor's 500 Index Calculation*, http://www.cftech.com

(v) *The Uses and Limits of Volatility*, David Harper, February 2004

(vi) *Index Design and Implication for Index Tracking: Evidence from S&P 500 Index Funds*, Alex Frino, David R. Gallagher, Albert S. Neubert, Teddy N. Oetomo, 2003

(vii) *Guide to Calculation Methods for the UK Series of the FTSE Actuaries Share Indices*, Version 4.1 May 2005

(viii) *SSE 180 and SSE 50 Index Methodology*, China Securities Index, September 2007

(ix) *Rules for the HS60 Index*, HOLT, Version 4, October 2006

(x) *Index Methodology and Calculation,* Bakers Investment Group January 2008

Appendix 1: Formula for divisor adjustment

(1) Proof : Equally weighted option price average index

Assuming at the time T+1, prices of all remaining (or that, not-yet-expired) options unchanged, we should proof the index will also remain the same. Under this assumption, we have $\sum(\mathrm{Re}\,main)|_{t+1} = S_t - \sum(Expired)|_{(t)}$ (here S_t is the sum of options prices at time T), so

$$S_{t+1} = \sum(\mathrm{Re}\,main)|_{t+1} + \sum(New)|_{(t+1)} = S_t - \sum(Expired)|_{(t)} + \sum(New)|_{(t+1)} = S_t^{(a)}$$

Since $d_{t+1} = d_t \dfrac{S_t^{(a)}}{S_t} / \dfrac{N_{t+1}}{N_t}$, we have

$$I_{t+1} = \frac{S_{t+1}}{d_{t+1}N_{t+1}} = \frac{S_{t+1}}{N_{t+1}} / \left(d_t \frac{S_t^{(a)}}{S_t} / \frac{N_{t+1}}{N_t} \right) = \frac{S_t^{(a)}}{N_{t+1}} / \left(d_t \frac{S_t^{(a)}}{S_t} / \frac{N_{t+1}}{N_t} \right) = \frac{S_t}{d_t N_t} = I_t$$

Therefore, in this case, the option price average index at time T+1 remains at the same as at time T.

(2) Proof : non-equally-weighted option price average index

Let’s suppose it is weighted by Volume (V). Similarly, assuming prices and volumes for all remaining options at time T+1 not changed, we should proof the weighted option price index also unchanged. In this case, we have

$$\sum V_{(k,t+1)} P_{(k,t+1)} |_{remaining} = S_t - \sum_{j=1}^{N(e,t)} V_{\mathrm{exp}ired(j,t)} P_{epired(j,t)} \quad \text{(here, } S_t = \sum V_{(i,t)} P_{(i,t)} |_{all} \text{)}$$

$$\sum V_{(k,t+1)} |_{remaining} = V_t - \sum_{j=1}^{N(e,t)} V_{\mathrm{exp}ired(j,t)} \quad \text{(here, } V_t = \sum V_{(i,t)} |_{all} \text{)}$$

So

$$S_{t+1} = \sum V_{(k,t+1)} P_{(k,t+1)} |_{remaining} + \sum_{j=1}^{N(n,t+1)} V_{new(j,t+1)} P_{new(j,t+1)}$$

$$= S_t - \sum_{j=1}^{N(e,t)} V_{\mathrm{exp}ired(j,t)} P_{epired(j,t)} + \sum_{j=1}^{N(n,t+1)} V_{new(j,t+1)} P_{new(j,t+1)} = S_t^{(a)}$$

$$V_{t+1} = \sum V_{(k,t+1)} |_{remaining} + \sum_{j=1}^{N(n,t+1)} V_{new(j,t+1)} = V_t - \sum_{j=1}^{N(e,t)} V_{\mathrm{exp}ired(j,t)} + \sum_{j=1}^{N(n,t+1)} V_{new(j,t+1)} = V_t^{(a)}$$

Since $WPAI_{t+1} = \dfrac{S_{t+1}}{d_{t+1}V_{t+1}} = \dfrac{S_t^{(a)}}{d_{t+1}V_t^{(a)}}$, and $d_{t+1} = d_t \left(\dfrac{S_t^{(a)}}{S_t} \right) / \left(\dfrac{V_t^{(a)}}{V_t} \right)$

Therefore,

$$WPAI_{t+1} = \frac{S_{t+1}}{d_{t+1}V_{t+1}} = \frac{S_t^{(a)}}{d_{t+1}V_t^{(a)}} = \frac{S_t^{(a)}}{V_t^{(a)}} / \left[d_t \left(\frac{S_t^{(a)}}{S_t} \right) / \left(\frac{V_t^{(a)}}{V_t} \right) \right] = \frac{S_t}{d_t V_t} = WPAI_t$$

Appendix 2: Correlations and Volatilities

Tick	Correlation Coefficient with Underlying Stock		Annualized Volatilities		Annualized Volatilities of Underlying Stock
	COPAIs	**POPAIs**	**COPAIs**	**POPAIs**	
AA	0.8462	-0.8853	70.1%	68.8%	29.9%
AIG	0.7226	-0.9774	53.9%	44.1%	24.7%
AXP	0.9092	-0.5195	43.7%	40.9%	26.2%
BA	0.9489	-0.8442	50.7%	61.3%	22.5%
C	0.9330	-0.9761	62.3%	34.6%	25.8%
CAT	0.3381	0.2629	75.0%	64.9%	45.9%
DD	0.7979	-0.8565	49.9%	36.8%	20.3%
DIS	0.8977	-0.9584	37.4%	36.4%	19.6%
GE	0.7616	-0.8696	43.0%	41.3%	16.6%
GM	0.7890	-0.7697	49.8%	50.9%	42.5%
HD	0.8620	-0.9258	57.0%	38.8%	24.3%
HON	0.9697	-0.8933	43.6%	53.0%	20.9%
HPQ	0.9626	-0.8704	51.5%	64.8%	26.1%
IBM	0.8952	-0.9717	43.5%	38.8%	18.9%
INTC	0.8304	-0.9328	73.4%	33.6%	27.7%
JNJ	0.7124	-0.8297	35.1%	29.1%	12.5%
JPM	0.9249	-0.9522	51.5%	45.2%	24.6%
KO	0.9852	-0.9457	31.1%	35.3%	13.2%
MCD	0.9663	-0.9126	49.4%	52.0%	20.9%
MMM	0.6979	-0.8328	54.8%	43.3%	18.3%
MO	0.6532	-0.4175	58.9%	96.5%	71.0%
MRK	0.9649	-0.9379	66.4%	59.6%	24.8%
MSFT	0.8870	-0.9216	37.8%	40.5%	21.0%
PFE	0.7990	-0.9228	53.5%	32.7%	20.5%
PG	0.9669	-0.9508	26.1%	29.7%	14.2%
T	0.9898	-0.9026	49.6%	58.2%	19.7%
UTX	0.3055	0.1071	42.4%	37.5%	42.8%
VZ	0.9227	-0.9173	35.6%	42.5%	18.4%
WMT	0.5799	-0.7640	58.4%	40.0%	18.3%
XOM	0.9727	-0.9314	50.5%	59.6%	22.8%
Average	**0.8264**	**-0.8007**	**50.2%**	**47.0%**	**25.2%**